Concept Analyst

VERIFICATION EXAMPLES

Version 1.8

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Contents

DISCLAIMER

This software is provided as a tool for an engineer. While it has undergone a series of tests, it is important to recognise that it is performing numerical simulations of physical phenomena, and that these involve approximations. An engineer performing these simulations with Concept Analyst does so at his/her own risk. Neither Concept Analyst, Ltd. nor the University of Durham shall be held responsible for the results obtained, nor for any consequential loss. Confirming the accuracy and/or usefulness of all the solutions is the responsibility of the licensee or user.

1

Introduction

This document presents some example problems that engineers may use as a basis to verify the results of the Concept Analyst software. For each example, the following shall be presented:

- A description of the geometry and loading
- Result(s) taken from another source
- Concept Analyst results

Users are encouraged to test their own installation of Concept Analyst using these and/or other examples in order to gain confidence in the results and to gain some intuition into the accuracy that is to be expected from different types of problem with different types of mathematical model.

Concept Analyst has been tested extensively. In each example there are usually several parameter combinations available (such as length, radii etc). In each case, these parameters are varied, and scaled versions of each model are subsequently tested, providing several hundred models for each example. For each example just a small sample of results is presented for verification.

2

Plate with hole

This is a classical stress concentration problem. A rectangular plate containing a central hole is subjected to uniaxial stress in the horizontal direction (Figure 2-1).

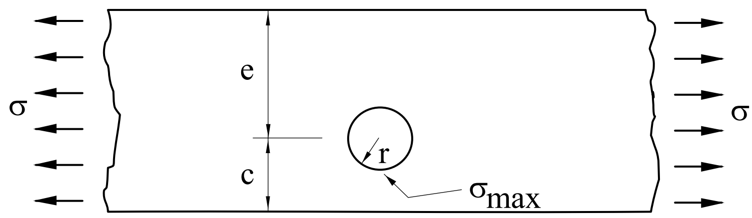

Figure 2-1. Plate with hole

The stress concentration gives rise to a peak stress at the bottom of the circular hole, as shown. The solutions of Peterson[1] are used to form a comparison.

Multiple combinations of these parameters defined in the figure (as well as scaled versions of each model) have been used for verification. Four sample cases are presented in this section. For the Concept Analyst model of this geometry, it is necessary to make some assumption about the length of the plate, since it is clearly

[1] R.E. Peterson, Stress Concentration Factors, Wiley, 1974.

impossible to sketch and analyse an infinite strip using the facilities in the program. Therefore the plate length, L, is an additional parameter that has been considered. For the results presented, a plate of 12 mm width is used.

A Concept Analyst model for this case is as follows:

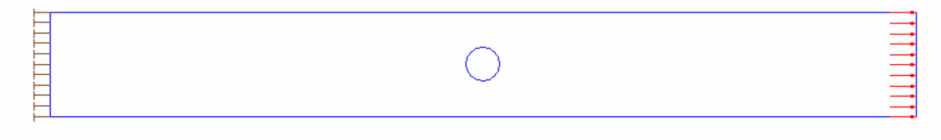

Figure 2-2. Concept Analyst model

Notice that no constraint is applied in the vertical direction. The program will apply a soft spring constraint (see the Concept Analyst User Guide) and this generally provides the most accurate results for problems exhibiting incomplete constraint. In other words, any constraint applied in the vertical direction would be changing the conditions under which the plate is loaded, and would therefore tend to invalidate the comparison.

Sample results are presented for fine, standard and coarse mesh density settings, in the form of stress concentration factors, K_t, calculated from the maximum principal stress, σ_1 (figure 3). Also presented is a comparison of Concept Analyst results in the format found in Peterson (figure 4).

L/mm	r/c	e/c	K_t Peterson	K_t Fine Mesh	K_t Standard Mesh	K_t Coarse Mesh	K_t Coarse Adaptive
50	0.1	1	3.04	3.036	3.037	3.025	3.025
100	0.5	1	4.30	4.241	4.252	4.134	4.288
50	0.3	2	3.29	3.320	3.297	3.293	3.320
100	0.5	2	4.14	4.194	4.122	4.106	4.111

Figure 2-3. Selected Concept Analyst Results for plate of width 12mm

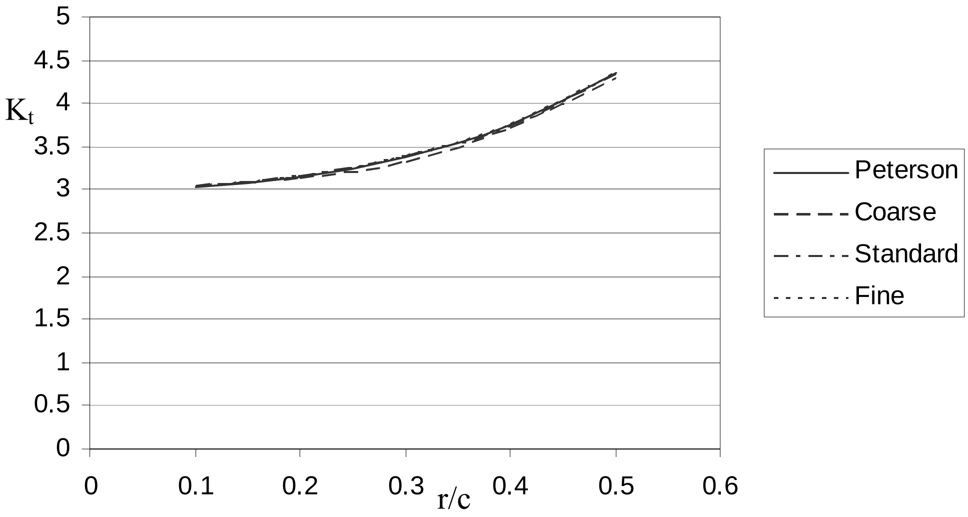

Figure 2-4. Sample Concept Analyst (L=100mm) results compared to Peterson (L=∞) for e/c = 1

Notes:

1. Some of the difference between Peterson's results and those of Concept Analyst is due to the fact that the infinite plate has been approximated by one of finite length. The comparison can be observed to be closer if a longer plate is used.

2. The Peterson results in this example are read from a graph and can be interpreted only within a coarse resolution.

3. The Concept Analyst results presented are those taken from the maximum level on an *x-y* boundary graph plot of maximum principal stress.

The examples given in Figure 2-3 have been used to verify the option to specify boundary conditions as non-zero displacements. By taking displacements found on the right-hand side edge of the model and applying them as non-zero displacement boundary conditions, the resulting stress values along the right-hand edge of the new model should equal the load applied in the original model.

An x-y plot of the x-displacement of the right-hand side of the model (Figure 2-2) gives the displacement values for the new model (Figure 2-5), where the load boundary condition as been replaced with a non-zero displacement boundary condition.

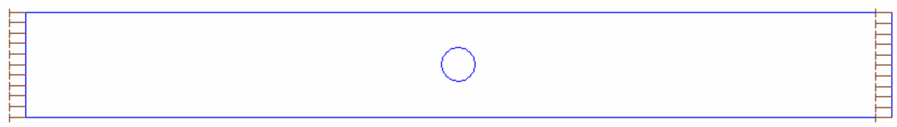

Figure 2-5. Concept Analyst model

Let *u* denote the *x*-displacement of the right-hand side nodes of the original model using the material properties for mild steel. *u* varies slightly over the length of the line, and a mean value is applied now as a boundary condition in place of the traction that was previously applied to generate the results in Figure 2-3. Figure 2-6 shows the peak value of maximum principal stress on the hole perimeter using non-zero displacement boundary conditions. Coarse, standard and fine mesh settings were used. This peak stress is compared with the value predicted by Peterson for the original set of boundary conditions (an applied load) as for Figure 2-3.

L/mm	r/c	e/c	σ_1 max Peterson	*u*/mm Coarse	σ_1 max Coarse	*u*/mm Standard	σ_1 max Standard	*u*/mm Fine	σ_1 max Fine
50	0.1	1	304	0.02429	302.5	0.02429	303.7	0.02429	303.6
100	0.5	1	430	0.05281	413.0	0.05291	425.0	0.05292	423.9
50	0.3	2	329	0.024745	325.7	0.024745	326.2	0.024745	328.3
100	0.5	2	414	0.050185	400.1	0.050185	402.8	0.05020	411.1

Figure 2-6. Concept Analyst Results for non-zero displacement boundary condition problems (Standard Mesh). Stresses in MPa.

3

Plate with uneven holes

This example is also a comparison with results from Peterson. It involves the stress concentrations around two holes of different diameter in an infinite plate under a uniaxial stress field (Figure 3-1).

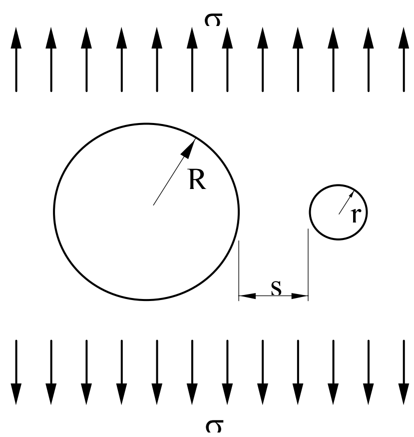

Figure 3-1. Infinite plate with two uneven holes

For the Concept Analyst model of this geometry, it is necessary to make some assumption about the size of the plate, since it is clearly impossible to sketch and analyse an infinite plate using the facilities in the program.

A Concept Analyst model for this case is as follows:

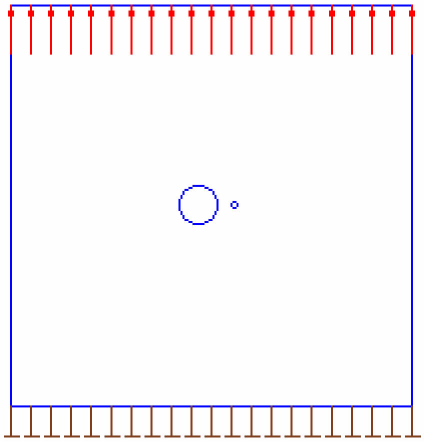

Figure 3-2. Concept Analyst model

Notice that no constraint is applied in the horizontal direction. The program will apply a soft spring constraint (see the Concept Analyst User Guide) and this generally provides the most accurate results for problems exhibiting incomplete constraint. In other words, any constraint applied in the horizontal direction would be changing the conditions under which the plate is loaded, and would therefore tend to invalidate the comparison.

Sample results are presented for fine, standard and coarse mesh density settings, in the form of stress concentration factors, K_t, calculated from the maximum principal stress, σ_1 (figure 3-3). In these examples, a plate of dimensions 10 x 10 mm contains two holes of radius R and r. Also presented is a comparison of Concept Analyst results in the format found in Peterson (figure 3-4).

R/r	R/mm	s/r	K_t Peterson	K_t Fine Mesh	K_t Standard Mesh	K_t Coarse Mesh	K_t Coarse Adaptive
1	0.1	3	3.1	3.039	3.037	3.034	3.034
1	0.4	3	3.1	3.098	3.096	3.095	3.095
5	0.1	4	3.5	3.482	3.485	3.485	3.487
5	0.4	4	3.5	3.517	3.518	3.518	3.520
10	0.1	5	4.1	4.107	4.110	4.110	4.114
10	0.4	5	4.1	4.146	4.149	4.148	4.153

Figure 3-3. Selected Concept Analyst results for plate dimensions 10mm x 10mm where the centre of the plate coincides with the mid-point of dimension ‘s’ in Fig 3-1

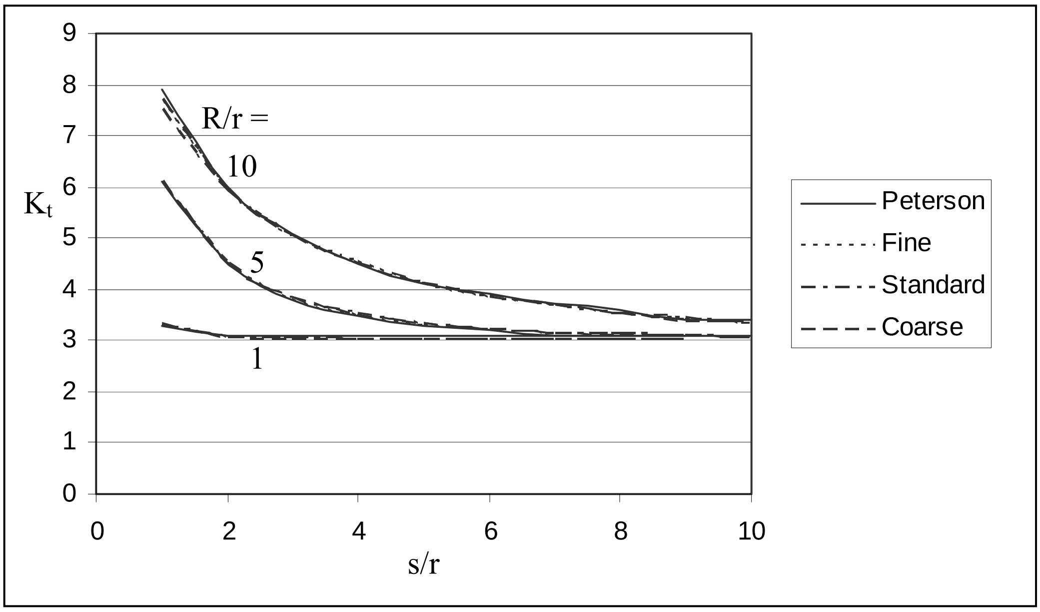

Figure 3-4. Sample Concept Analyst (length = 10mm) results compared to Peterson (length = ∞)

Notes:

1. Some of the difference between Peterson's results and those of Concept Analyst are due to the fact that the infinite plate has been approximated by one of finite length. The comparison can be observed to be closer if a longer plate is used.

2. The Peterson results in this example are read from a graph and can be interpreted only within a coarse resolution.

3. If the centre of the plate is not coincident with the mid-point of dimension 's' in Figure 3-1, the results in Figure 3-3 will be slightly different. This will be particularly the case in the models for which R = 0.4mm, i.e. the hole is no longer very small in comparison with the plate.

4

Rectangular hole with rounded corners

This is the third example to compare the results of Peterson with those of Concept Analyst. It involves the stress concentrations around a rectangular hole, with rounded corners, in an infinite plate under a biaxial tensile stress field (Figure 4-1).

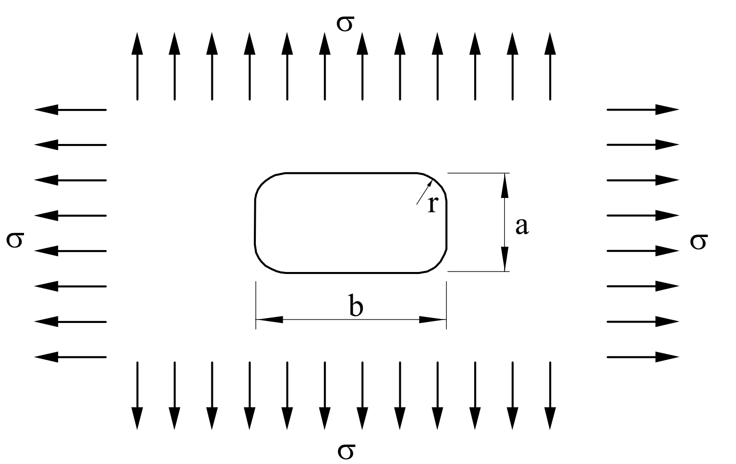

Figure 4-1. Rectangular hole with rounded corners

For the Concept Analyst model of this geometry, it is necessary to make some assumption about the size of the plate, since it is clearly impossible to sketch and analyse an infinite plate using the facilities in the program.

A Concept Analyst model for this case is as follows:

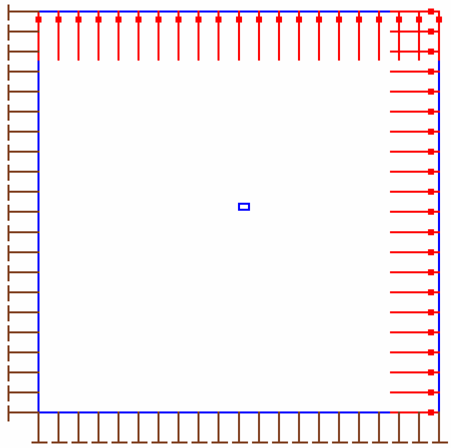

Figure 4-2. Concept Analyst model

Sample results are presented for fine, standard and coarse mesh density settings, in the form of stress concentration factors, K_t, calculated from the maximum principal stress, σ_1 (figure 4-3). In these examples, a plate of dimensions 10 x 10 mm contains a hole of varying b- and r-dimensions, where dimension a = 0.1 mm. Also presented is a comparison of Concept Analyst results in the format found in Peterson (figure 4-4).

b/a	r/b	K_t Peterson	K_t Fine Mesh	K_t Standard Mesh	K_t Coarse Mesh	K_t Coarse Adaptive
1.0	0.10	4.87	4.826	4.860	4.860	4.859
1.5	0.12	4.11	4.068	4.062	4.062	4.061
2.0	0.17	3.23	3.298	3.307	3.307	3.298
2.5	0.10	4.21	4.177	4.179	4.178	4.177
3.0	0.14	3.68	3.631	3.592	3.592	3.630

Figure 4-3. Selected Concept Analyst results for plate dimensions 10mm x 10mm

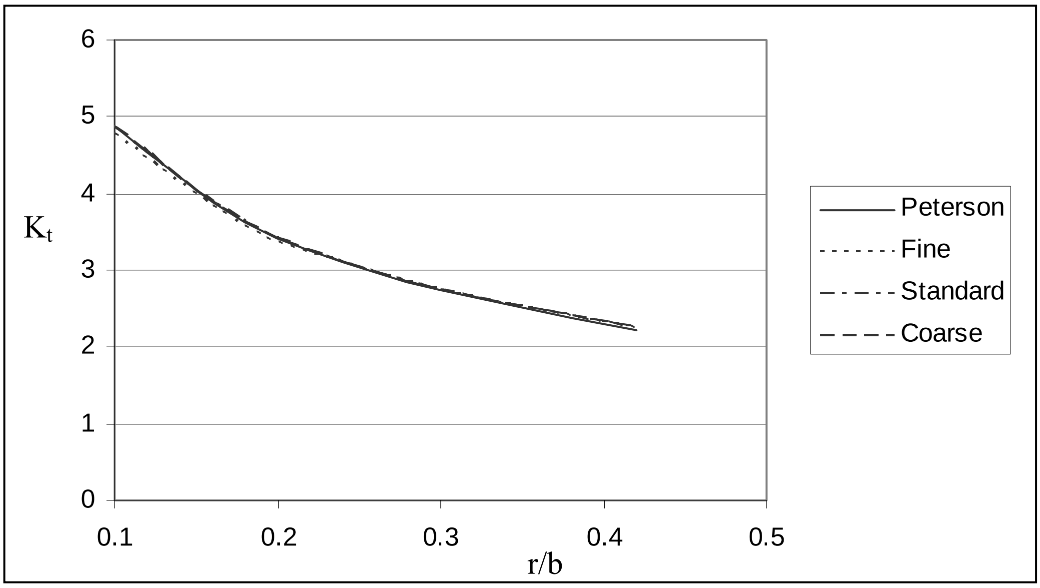

Figure 4-3. Selected Concept Analyst (length = 10mm) results compared to Peterson (length = ∞) where b/a = 1

Notes:

1. Some of the difference between Peterson's results and those of Concept Analyst are due to the fact that the infinite plate has been approximated by one of finite length. The comparison can be observed to be closer if a longer plate is used.

2. The Peterson results in this example are read from a graph and can be interpreted only within a coarse resolution.

5

Beam in bending

This example presents a comparison with the theoretical deflections of beams in bending. Many comparisons of this type are complicated because of the combined bending and shear effects that comprise the total bending deflection. In this case, a pure bending moment is applied, with no shear, and therefore a clear comparison may be made with theoretical solutions (figure 5-1).

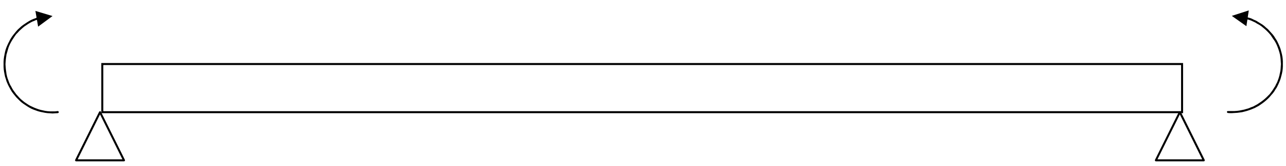

Figure 5-1. Beam in bending

In this section, Concept Analyst is used to model half of the free-free beam to enable simple determination of the tip deflection. Therefore, the 'beam length' in the table below should be considered to be L/2 when comparing with the theoretical formula.

The Concept Analyst model is as follows:

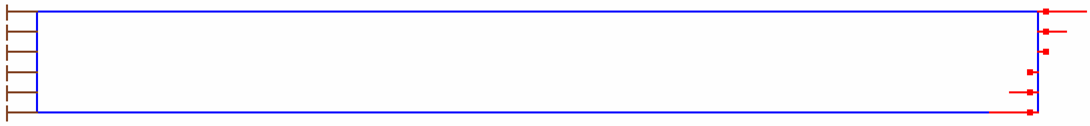

Figure 5-2 Concept Analyst model

The load is applied using a non-uniform x-direction distributed load over the right hand edge. This applies a pure bending moment. Constraint is applied only in the x-direction, a soft spring constraint being automatically applied by the program in the y-direction. This is important in maintaining a condition of pure bending throughout the geometry.

The theoretical deflection, v, for a free-free beam of length L under bending moment M is given by Benham, Crawford and Armstrong[2], and is

$$v = \frac{ML^2}{8EI}$$

where E is Young's Modulus and I is the second moment of area of the section.

In the following examples, a beam of constant depth 10 mm is varied in length. . Over the right hand edge of the plate, a non-uniform x-direction distributed load varied from 100N/mm^2 to –100N/mm^2. This loading may readily be shown to provide a pure bending moment of 1666.7Nmm. Concept Analyst's default plate/beam thickness of 1mm and its Mild Steel properties (E = 207000 N/mm^2) are used.

Sample results are presented for fine, standard and coarse mesh density settings, in the form of stress concentration factors, K_t, calculated from the maximum principal stress, σ_1 (figure 5-3). Also presented is a comparison of Concept Analyst results in graphical format (figure 5-4).

Beam length	Theory	Fine	Standard	Coarse	Coarse Adaptive
10	0.005	0.005	0.005	0.005	0.005
20	0.019	0.019	0.019	0.019	0.019
50	0.121	0.121	0.121	0.121	0.121
100	0.483	0.483	0.483	0.486	0.486
200	1.932	1.932	1.937	1.937	1.937
400	7.730	7.728	7.728	7.728	7.728

Figure 5-3. Concept Analyst displacement results (mm) for beam of depth 10mm

[2] P.P.Benham, R.J.Crawford & C.G.Armstrong, Mechanics of Engineering Materials, 2nd. Ed, Longman, 1996.

Notice that, because no vertical constraint is applied to the model, and a soft spring constraint used automatically in this direction, then neither the left end nor the right end of the beam has a zero displacement. The deflections produced by Concept Analyst are generated by considering the difference between the displacements at the two ends of the beam.

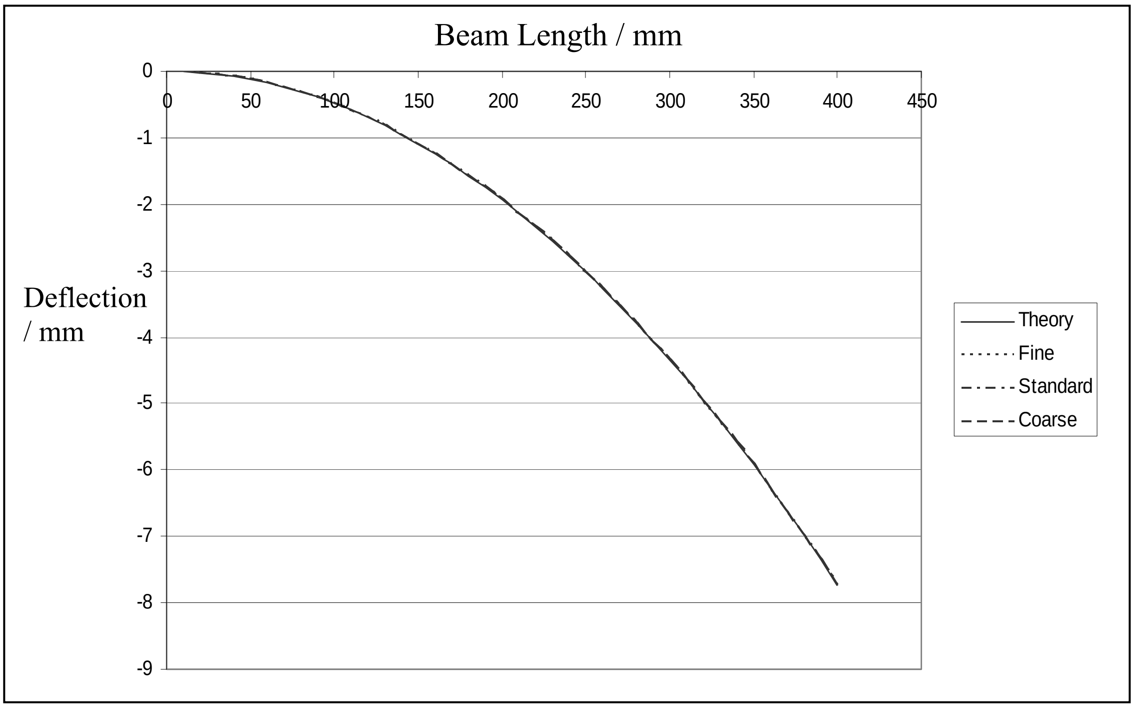

Figure 5-4. Selected Concept Analyst results compared to theoretical results for beam of depth 10mm

This shows very good correlation with theory. The stress distributions in all cases reflect accurately the pure bending boundary condition applied.

6

Thick-walled cylinder

This example compares the Concept Analyst results with the theoretical equations of elasticity in internally pressurised thick-walled cylinders. One quarter of the cylinder is to be considered (figure 6-1).

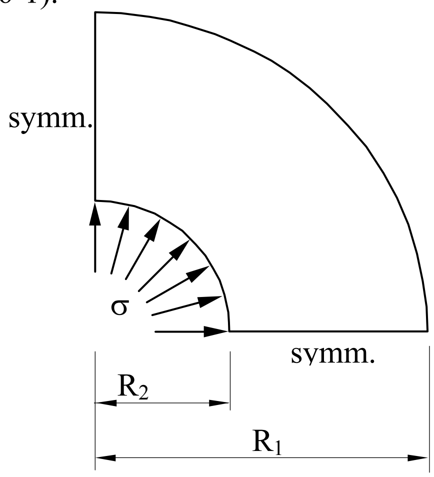

Figure 6-1. Thick walled cylinder

Concept Analyst is used to model a symmetrical section with an internal pressure of 100 MPa (Figure 6-2).

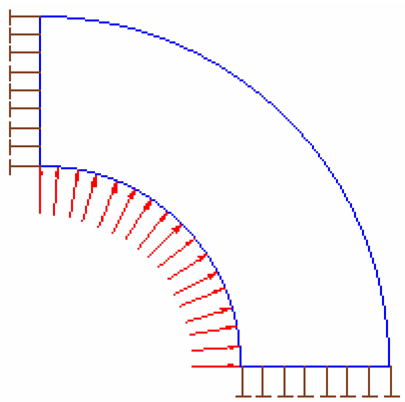

Figure 6-1. Concept Analyst model

Thick cylinder theory states that at any radial coordinate, r, the radial and hoop stress components, σ_θ and σ_r respectively, are given by the Lamé equations

$$\sigma_\theta = A + \frac{B}{r^2} \qquad \sigma_r = A - \frac{B}{r^2}$$

where A and B are constants for any cylinder/pressure, and these can be determined from the particular boundary conditions. In this case, the boundary conditions are zero radial stress at the outer radius r = R_1 and a radial stress equal to the internal pressure 100 MPa at the inner radius r = R_2. These give constants A and B. The maximum stress in the cylinder is the hoop stress at the inner radius, and this may be readily determined from the above equation by substituting values of r and determining constants A and B in each case.

Sample results are presented for fine, standard and coarse mesh density settings, in the form of stress concentration factors, K_t, calculated from the maximum principal stress, σ_1 (figure 6-3). Also presented is a comparison of Concept Analyst results in graphical format (figure 6-4).

R_1	R_2	K_t Theory	K_t Fine Mesh	K_t Standard Mesh	K_t Coarse Mesh	K_t Coarse Adaptive
100	50	1.667	1.667	1.667	1.668	1.667
200	50	1.133	1.134	1.134	1.138	1.135
300	100	1.250	1.250	1.251	1.251	1.253
475	100	1.093	1.093	1.094	1.097	1.094
788	175	1.104	1.104	1.105	1.108	1.105

Figure 6-3. Selected Concept Analyst results

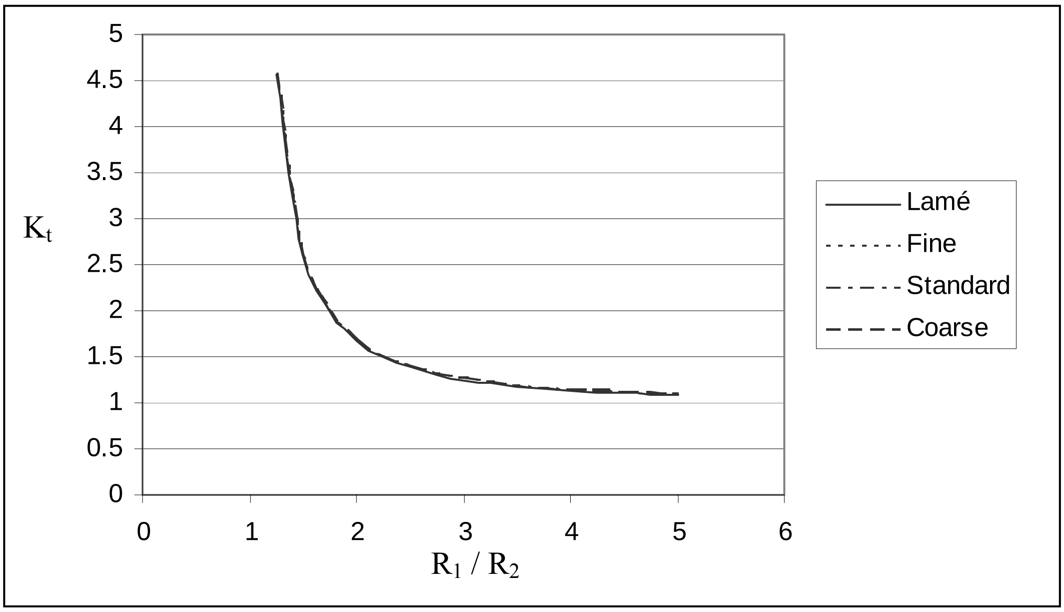

Figure 6-4. Selected Concept Analyst results compared to Lamé results

This shows very good correlation with theory.

7

Plate with opposite notches

This example is also a comparison with results from Peterson. It involves the stress concentrations around two notches of opposite identical radii in an infinite plate under a uniaxial stress field (Figure 7-1).

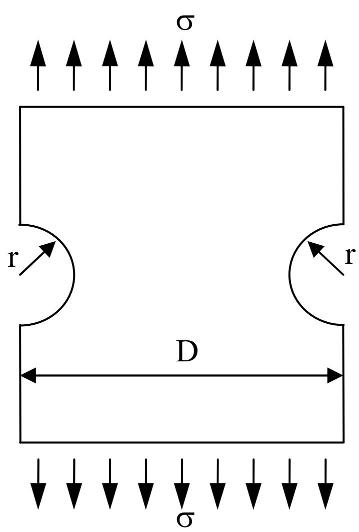

Figure 7-1. Plate with opposite semi-circular notches

For the Concept Analyst model of this geometry, it is necessary to make some assumption about the size of the plate, since it is clearly impossible to sketch and analyse an infinite plate using the facilities in the program.

A Concept Analyst model for this case is as follows:

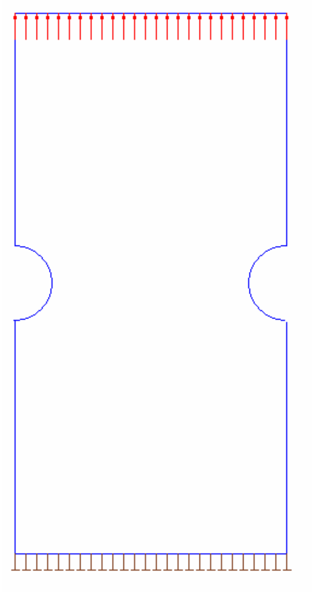

Figure 7-2. Concept Analyst model

Notice that no constraint is applied in the horizontal direction. The program will apply a soft spring constraint (see the Concept Analyst User Guide) and this generally provides the most accurate results for problems exhibiting incomplete constraint. In other words, any constraint applied in the horizontal direction would be changing the conditions under which the plate is loaded, and would therefore tend to invalidate the comparison.

Sample results are presented for fine, standard and coarse mesh density settings, in the form of stress concentration factors, K_t, calculated from the maximum principal stress, σ_1 (figure 7-3). In these examples, a plate of 100 mm length and varying widths contains two semi-circular notches with a range of radii. Also presented is a comparison of Concept Analyst results in the format found in Peterson (figure 7-4).

D / mm	r / mm	K_t Peterson	K_t Fine Mesh	K_t Standard Mesh	K_t Coarse Mesh	K_t Coarse Adaptive
20	7	4.20	4.213	4.231	4.231	4.232
30	6	3.10	3.107	3.103	3.103	3.103
40	7	3.05	3.070	3.066	3.066	3.066
40	12	3.55	3.548	3.551	3.550	3.551
50	10	3.28	3.108	3.104	3.104	3.104

Figure 7-3. Selected Concept Analyst results for plate of length 100mm

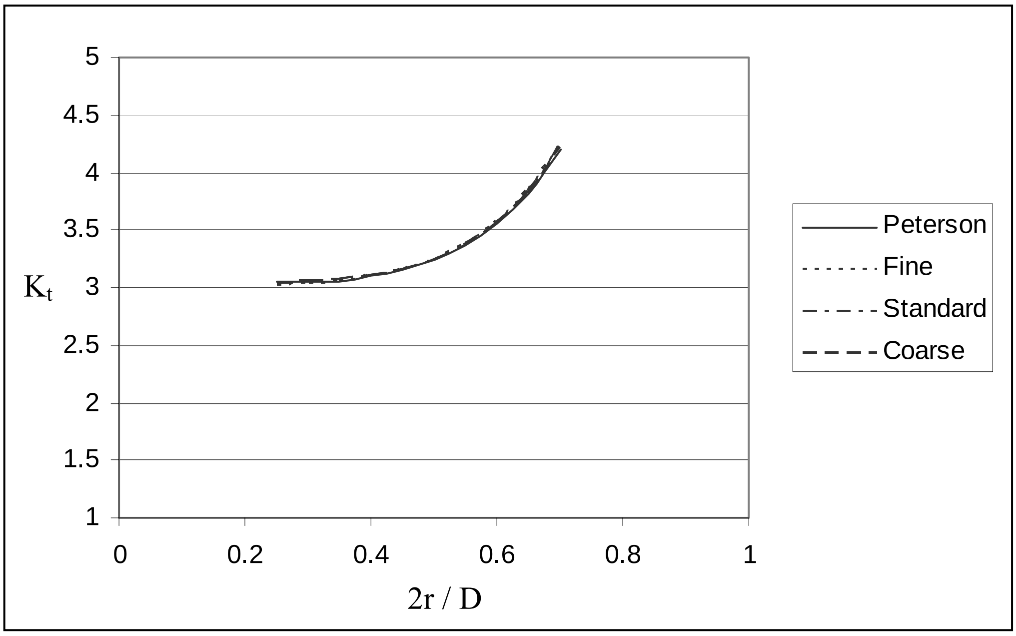

Figure 7-4. Selected Concept Analyst (length = 100mm) results compared to Peterson (length = ∞) for D = 40 mm

8

Plate with adjacent notches

This example is also a comparison with results from Peterson. It involves the stress concentrations around two adjacent notches of identical radii in an infinite plate under a uniaxial stress field (Figure 8-1).

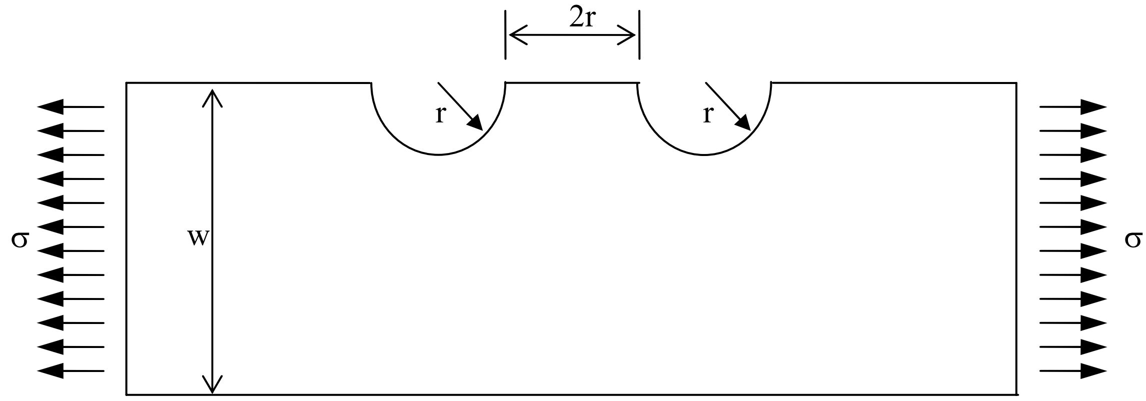

Figure 8-1. Plate with adjacent semi-circular notches

For the Concept Analyst model of this geometry, it is necessary to make some assumption about the size of the plate, since it is clearly impossible to sketch and analyse an infinite plate using the facilities in the program.

A Concept Analyst model for this case is as follows:

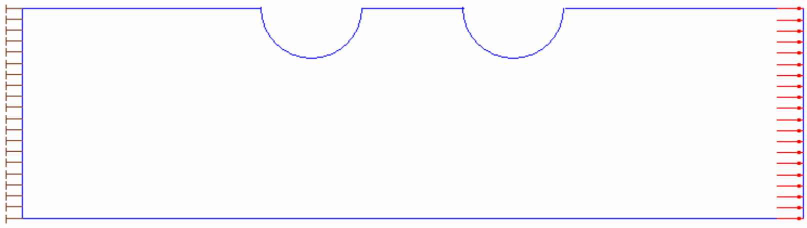

Figure 8-2. Concept Analyst model

Notice that no constraint is applied in the vertical direction. The program will apply a soft spring constraint (see the Concept Analyst User Guide) and this generally provides the most accurate results for problems exhibiting incomplete constraint. In other words, any constraint applied in the horizontal direction would be changing the conditions under which the plate is loaded, and would therefore tend to invalidate the comparison.

Sample results are presented for fine, standard and coarse mesh density settings, in the form of stress concentration factors, K_t, calculated from the maximum principal stress, σ_1 (figure 8-3). In these examples, a plate of 100 mm length and varying widths contains two semi-circular notches with a range of radii. Also presented is a comparison of Concept Analyst results in the format found in Peterson (figure 8-4).

w / mm	r / mm	w / r	K_t Peterson	K_t Fine Mesh	K_t Standard Mesh	K_t Coarse Mesh	K_t Coarse Adaptive
3	0.27	11.1	2.90	2.969	2.972	2.972	2.970
6	0.90	6.67	3.43	3.434	3.424	3.424	3.433
12	1.44	8.33	3.20	3.215	3.211	3.168	3.167
18	1.08	16.7	2.86	2.884	2.887	2.888	2.885

Figure 8-3. Selected Concept Analyst results of strip length 100mm

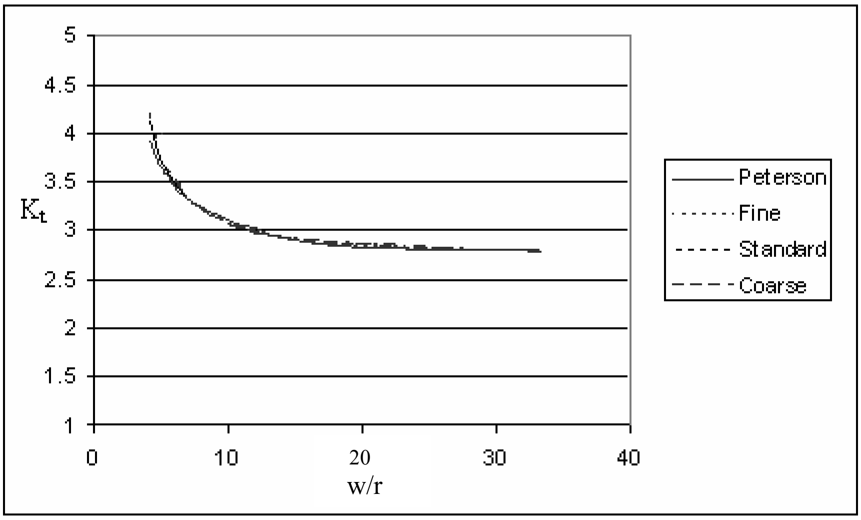

Figure 8-4. Selected Concept Analyst (length 100mm) results compared to Peterson (length = ∞) for w = 18 mm

Notes:

1. Some of the differences between Peterson's results and those of Concept Analyst are due to the fact that the infinite plate has been approximated by one of finite length. The comparison can be observed to be closer if a longer plate is used.

2. The Peterson results in this example are read from a graph and can be interpreted only within a coarse resolution.

9

Limitations

It should always be recognised that the results produced by Concept Analyst are an approximation to physical behaviour.

Like many forms of numerical approximation, the accuracy of the solution is directly related to the quality of the approximation, and this is in turn related to the time and effort invested in the approximation.

For programs like Concept Analyst, this essential trade-off is found in the definition of the ‘mesh’. Finite element and boundary element software systems all use elements (of one sort or another) to describe the geometry and results. It is usually the case that using more elements will give rise to better results, but will require more time and grater usage of computational resources than the coarser mesh.

Most analysis systems like this leave to the user this decision about the number of elements to use. This has the advantage that the user is free to undertake what is called a ‘convergence’ analysis, in which models of increasing complexity are run, and when the solutions do not change markedly from one run to the next, it may be assumed that convergence has been achieved and the numerical model is proving to be adequate.

On the other hand, leaving the decision to the user requires that user to be reasonably expert in the use of this technology. Concept Analyst takes the approach that these important decisions should be made automatically by the program according to some rules contained within the software algorithms (although a limited convergence analysis is still available through the use of the coarse, standard and fine mesh density settings).

This has the advantage of ease and speed of use, particularly in the hands of non-expert stress analysts. However, it is possible that a geometry might cause the program difficulties in automatically defining a suitable mesh of boundary elements for that particular problem